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P2_2 Attenborough Pyrotechnics.

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Abstract

In this paper we discuss the requirements of a firework, launched from the Attenborough Tower on The University of Leicester Campus, which would be capable of being viewed from any location in England. We calculated that the firework would need to reach a height of 14.3 km, requiring 7.79 g of gunpowder. The intensity of the explosion seen at the maximum distance from the Tower (Land's End) would be 8.07×10^{-13} W/m^2, which we found could be produced by 1.10 mg of Barium.

Firework Height

We started by using the distance from Leicester to Lands' End, 427 km [1], as the maximum distance from which the firework must be visible to the human eye. We needed to take into account the curvature of the Earth, found using Equation (1), where h is the height which is the drop in curvature over the distance, $r = 6371$ km is the radius of the Earth, and $a = 0.00899$ degrees is the angle between Leicester and Land's End.This is calculated by dividing the radius of Earth by its circumference. A diagram can be found in Figure 1.

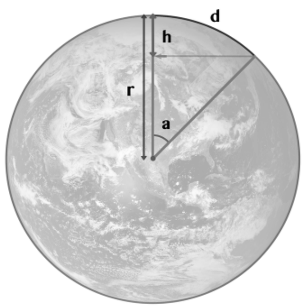

Figure 1: A labelled diagram of the Earth's curvature [3]

$$h = r(1 - cos(a)) \tag{1}$$

This means the firework must be at a height $h = 14.3$ km above the ground to be seen over the horizon. If we launched the rocket from the top of the Attenborough Tower on the University of Leicester campus (a height of 0.0520 km), it would need to explode 14.2 km into the air.

Firework Intensity

We also considered the brightness the firework would need to be visible against the night sky at this distance. The human eye is capable of seeing a magnitude of 6.5 and below [2]. We therefore used equation (2) to calculate the absolute magnitude, M, of the firework.

$$m - M = 5log(\frac{d}{10}) \tag{2}$$

Where $m = 6.5$ is the apparent magnitude and $d = 1.38 \times 10^{-11}$ pc is the distance to the observer in Land's End. This gave us the absolute magnitude $M = 65.8$, defined as the apparent magnitude of a light source at a distance of 10 pc. By rearranging Equation (3) for $L*$, the luminosity of the source, we can substitute this into Equation (4) to find an expression, shown in Equation (5) for the intensity of the firework at some distance, r.

$$M = -2.5log(\frac{L^*}{L_{\ sun}}) \tag{3}$$

$$I = \frac{L^*}{4\pi d^2} \tag{4}$$

$$I = \frac{10^{\frac{M}{-2.5}}}{4\pi r^2} \times L_{sun} \tag{5}$$

Where $M = 65.8$, $r = 427 \times 10^3$ m is the distance to the observer, and $L_{sun} = 3.84 \times 10^{26}$ W is the luminosity of the sun. This gave us a value for the intensity at Lands' End, the furthest point the firework will be visible, of 8.07×10^{-13} W/m^2.

Firework Constituents

From literature [4] we know 50 mg of gunpowder will launch a standard firework 91.4 m into the air. If we assume this relationship scales linearly, to reach the required height of 14.2 km our firework would need to be propelled by 7.79 g of gunpowder. In addition, as the human eye does not see all wavelengths equally, the firework would have to be green as this has been found to be the colour which is picked out most easily in the dark [6].

Barium powder is commonly used in fireworks as a source of green light, emitting wavelengths of 511-533 nm. If we take the average of 522 nm as the wavelength of the firework's light, we can the photon energy equation

$$E = \frac{hc}{\lambda} \tag{6}$$

to calculate the energy of one photon released by Barium is equal to 3.81×10^{-19} J, where c is the speed of light in a vacuum and h is Planck's Constant. If we assume one photon is emitted per Barium atom, we can calculate that to produce the energy needed to achieve a high enough luminosity for one second, we would need 5.09×10^{18} atoms, or 1.10 mg of Barium.

Assumptions

We have assumed there are no mountains or large buildings in the line of sight of the firework between Leicester and Land's End. Atmospheric effects have been ignored and we have assumed a dark background with no light pollution. We have assumed the amount of gunpowder needed to propel the firework scales linearly with height. For the calculation of mass of Barium required, we have assumed one photon is emitted per Barium atom and the explosion occurs over a timescale of 1 second.

Conclusion

We calculated the height a firework would need to reach to be seen across the whole of England, when fired from the top of the Attenborough Tower, is 14.3 km. This would require 7.79 g of gunpowder. The firework would have a minimum intensity of 8.07×10^{-13} W/m^2 at Lands End, the furthest point from the source. Green light generated by 1.10 mg of Barium would be the most appropriate colour choice, due to the human eye seeing this colour best in the dark.

References

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